Testing a dynamic dual-process model of inter-temporal choice: Serial and quasi-parallel processing

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Overview

- Motivation and Background
- Basic assumptions of the dynamic dual process model framework
- Serial processing: Two-stage dynamic dual process model
- Qualitative predictions
- Parallel processing: Parallel dynamic dual process model
- Qualitative predictions
- Implementing the Lowenstein et al. 2015 for intertemporal choice into framework
- Quantitative predictions
- Model test on Zhao et al. (2019) data

Dual process models

- Long tradition in psychology
- More recently in neuroscience

Dual process models

Applied to cognitive processes including reasoning and judgments

J.St.B.T. Evans (2008)

References	System 1	System 2
Fodor (1983, 2001)	Input modules	Higher cognition
Schneider & Schiffrin (1977)	Automatic	Controlled
Epstein (1994), Epstein & Pacini (1999)	Experiential	Rational
Chaiken (1980), Chen & Chaiken (1999)	Heuristic	Systematic
Reber (1993), Evans & Over (1996)	Implicit/tacit	Explicit
Evans (1989, 2006)	Heuristic	Analytic
Sloman (1996), Smith & DeCoster (2000)	Associative	Rule based
Hammond (1996)	Intuitive	Analytic
Stanovich (1999, 2004)	System 1 (TASS)	System 2 (Analytic)
Nisbett et al. (2001)	Holistic	Analytic
Wilson (2002)	Adaptive unconscious	Conscious
Lieberman (2003)	Reflexive	Reflective
Toates (2006)	Stimulus bound	Higher order
Strack & Deustch (2004)	Impulsive	Reflective

Dual process models, more recently

- Loewenstein et al. 2015: Framework for **intertemporal** choice, **risky** decisions, and **social preferences**
  - Affective system
  - Deliberate system
- Most popular since Kahneman (2011), Thinking, fast and slow
  - **System 1** Intuitive (fast, emotional, biased response, ...)
  - **System 2** Deliberate (slow, rational, normative response ...)

Dual process model in neuroscience

Human mind is composed of of multiple systems that approach decision in distinct ways

- Fast, habit-based system
- Slow, goal directed system

Reviews: Dolan & Dayan, 2013; Rangel et al. 2008

Problems of most dual process approaches

- Verbal – allows no quantitative predictions
- Unclear about processing
- Reverse inference

For the few **formal** models (Loewenstein et al. 2015, Mukherjee, 2010):

- No time mechanism
- (Unclear about processing)

Dynamic dual process model

- Diederich, A. & Trueblood, J.T. (2018). A dynamic dual process model of risky decision making. *Psychological Review*, 125(2), 270 – 292.
- Diederich, A. (2024) A Dynamic Dual Process Model for Binary Choices: Serial Versus Parallel Architecture. *Computational Brain & Behavior*, 7:37–64.

Intertemporal choice

Choices between rewards and punishments at different points in time

- Amount x_1 immediately
- Amount x_t at time t
- $x_1 < x_t$

System 1 has a tendency to choose immediate rewards over delayed rewards.
This tendency may be modified by the subsequent System 2.

Dynamic stochastic model: Basic assumptions

- Consequences of choosing each option are compared continuously over time $\rightarrow$ preferences are constructed
- Preference accumulation process with preference update
- Random fluctuation in accumulating preference strength

Preference Process

- Option R_I: immediate reward
- Option R_D: delayed reward
- $P(t)$: relative preference strength for choosing one option (R_I) over another option (R_D) at time t
- $P(t) < 0$: momentarily favoring R_I
- $P(t) > 0$: momentarily favoring R_D
- $V_{R_I}(t)$: momentary valence for the immediate reward
- $V_{R_D}(t)$: momentary valence for the delayed reward

Preference Process

- Preference strength is updated from one moment, t, to the next, $(t+\tau)$ by an input valence $V(t)$
- $V(t) = V_{R_D}(t) - V_{R_I}(t)$
- $V(t)$ reflects the momentary comparison of consequences produced by imagining the choice of either option R_I or R_D
- V(t) fluctuates

Preference Process and dual processes

- The input valence also depends on the system i, $i = 1, 2$ in which the DM operate
- The preference process is described by

$$P(t+\tau) = P(t) + V_i(t+\tau).$$

- Mean valence $E[V_i(t)] = \mu_i t$
- μ_i: **drift rate**, indicates the direction and strength of preference towards choosing option R_D or R_I
- $\mu_i > 0 \rightarrow R_D$
- $\mu_i < 0 \rightarrow R_I$

Stopping

The preference process stops and a final decision is made as soon as the preference state exceeds a decision threshold or boundary

- If $P(t) > \theta_{R_D} > 0$, the DM chooses the delayed reward R_D.
- If $P(t) < \theta_{R_I} < 0$, the DM chooses the immediate reward R_I.

Valence construction for inter-temporal choice situations

- System 1

$$V_1(t) = V_{1_{R_D}}(t) - V_{1_{R_I}}(t) \qquad \text{with drift rate } \mu_1$$

- System 2

$$V_2(t) = V_{2_{R_D}}(t) - V_{2_{R_I}}(t) \qquad \text{with drift rate } \mu_2$$

Magnitude of drift rates

- System 1

$$\mu_1 \begin{cases} > 0 & \text{in favor of choosing } R_D \\ = 0 & \text{indifferent between choosing } R_D \text{ and } R_I \\ < 0 & \text{in favor of choosing } R_I \end{cases}$$

- System 2

$$\mu_2 \begin{cases} > 0 & \text{in favor of choosing } R_D \\ = 0 & \text{indifferent between choosing } R_D \text{ and } R_I \\ < 0 & \text{in favor of choosing } R_I \end{cases}$$

Model architectures: 2 different views

- "**Default-interventionist**" approach: sequential ordering of the two systems with System 1 starting first and System 2 intervening later. This is the prevalent assumption in reasoning and preferential choice research with System 1 driven by heuristics and mainly responsible for biases (e.g., Evans 2008, Kahneman, 2011)
- "**Parallel-competitive**" approach: systems operate in parallel and are competitive (Sloman, 1996).

Model architectures

- For **each system** a **different** accumulation **process** takes place.
- **Case 1**: Attention switches from one system to the other system, and the two systems are processed **serially** $\rightarrow$ **Two-stage dynamic dual process model**
- **Case 2**: Attention switches between the two systems, and the two systems are processes in **parallel** $\rightarrow$ **Parallel dynamic dual process model**

Predictions

- The models make several **qualitative** and **quantitative** predictions for choice probabilities and mean choice response times patterns depending on the relation between μ_1 and μ_2.
- Here: how the amount of time operating in System 1 affects possibles choice probabilities.

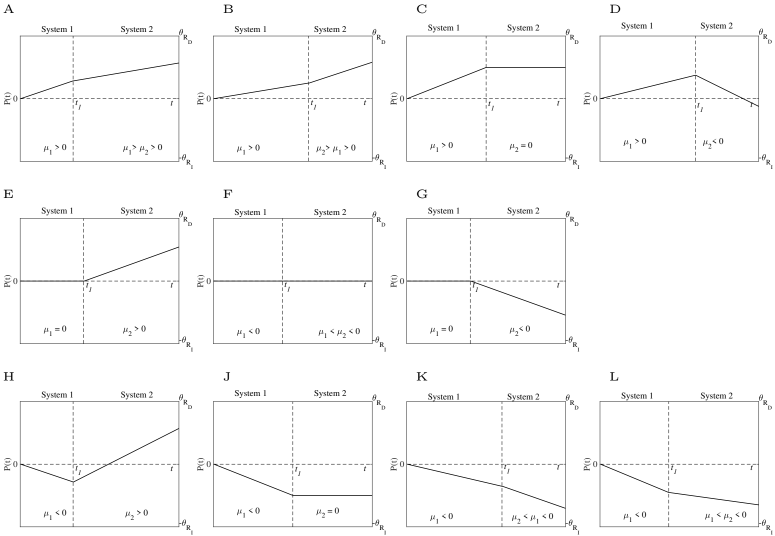
Myopic effect

- **Myopic behavior**: a preference for the immediate reward (Loewenstein et al., 2015)
- **Myopic effect**: magnitude thereof

Two-stage dynamic dual process model

- Attention switches from one system to the other system, and the two systems are processed **serially** $\rightarrow$ **Two-stage dynamic dual process model**

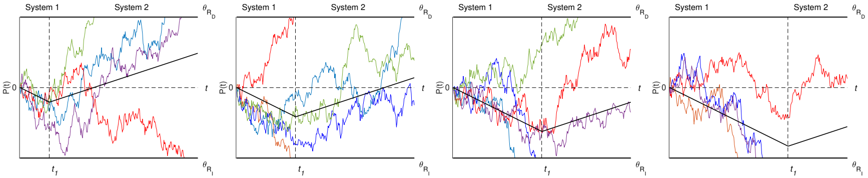
Two-stage dynamic dual process model



Note

- System 1 precedes System 2.
- Operating time in System 1 lasts t_1 time units before it switches to operating in System 2.
- Operating time may be a random variable T.
- In the following: μ_1 and μ_2 point not in the same direction.

Qualitative prediction



- The longer the DM operates in System 1, the stronger the myopic effect is because more and more decisions are made relying only on System 1.
- A preference reversal may occur as a function of operation time in System 1.

Note

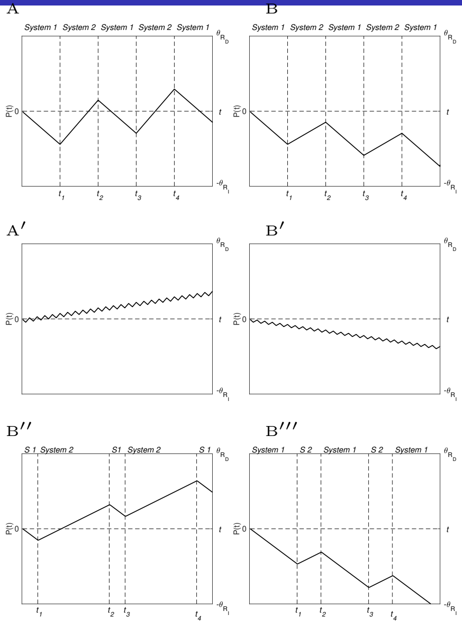
- Operations in System 1 indirectly influence System 2, with System 2's preference process starting where System 1's process ends.
- System 1 may influence System 2 by allowing the equations defining drift rate μ_2 to be a function of drift rate μ_1.

Parallel processing

- Most widely used: Collapse information of all processes, e.g. sum and map onto one drift rate μ
- Waited average or mixture of processes
- Independent race between processes
- (Dependent race between processes)
- Fast switching between processes $\rightarrow$ quasi parallel processing with dependency

Parallel dynamic dual process model

- Attention switches between the two systems, and the two systems are processes in **parallel** $\rightarrow$ **Parallel dynamic dual process model**



- The underlying distributions for T_j and their expected values are referred to as **time schedule**
- The sequence in which the systems are considered is called **order schedule**.
- Both time and order schedules are parameters of the model.

Basic quasi-parallel dual process model

- A model closely aligned with a parallel structure should assign an equal probability for each system to initiate the process.
  - The probability to start with System 1 and with System 2 is both 0.5.
  - The operating times in each system have the same distributions with identical parameters.

Qualitative prediction

- **Basic quasi-parallel model**: the operating times in System 1 and System 2 should have no or little effect on the size of the myopic effect.
- The larger drift rate (absolute value) determines the direction of the preference process towards the choice alternatives. Therefore, no preference reversals should occur as a function of operating times in both systems.
- Note: It is more difficult to make qualitative predictions for the remaining time and order schedules

Prelims for quantitative predictions

A simplified version of the model proposed by Loewenstein et al. (2015) for intertemporal choice.

- Model is static and deterministic
- Model is motivated as a dual-process approach (affective versus deliberate)
- Behavior is determined by a single "objective" function, $\mathcal{V}(R)$, of the rewards R

System 1 and System 2

(Loewenstein et al., 2015)

- System 1: $h(W, \sigma) \cdot M(R, a)$
  - Motivational function $M(R, a)$
  - Willpower strength and cognitive demands function $h(W, \sigma)$
- System 2: $U(R)$
- The model assumes that both processes operate simultaneously.
- The subjective value $\mathcal{V}(R)$ of an option is the sum of both processes.

$$\mathcal{V}(R) = U(R) + h(W, \sigma) \cdot M(R, a)$$

System 1

$$M(R, a) = R_I + \exp(-\delta_A(a) \cdot T_D) \cdot R_D$$

- T_D: time payoff R_D is received
- $\delta_A(a)$: discounting factor of an exponential discounting function for the affective system. a captures the intensity of affective motivations. Increased affective intensity a implies a smaller $\delta_A(a)$

$$h(W, \sigma)$$

- h reflects the willpower strength W and cognitive demands σ. The form of this function is not specified but is meant to be decreasing in W and increasing in σ.

System 2

$$U(R) = R_I + \exp(-\delta_D \cdot T_D) \cdot R_D$$

- δ_D is the discounting factor of an exponential discounting function for the deliberate system
- $\delta_A(a) < \delta_D$

Implementation into dynamic stochastic dual process framework: System 1

- $\delta_A(a) = \delta_1$
- $E[V_{1_I}(t)] = h(W,\sigma) \cdot R_I \cdot t$ for the immediate reward
- $E[V_{1_D}(t)] = h(W,\sigma) \cdot \exp(-\delta_1 \cdot T_D) \cdot R_D \cdot t$ for the delayed reward, resulting in a mean difference in valence (which defines the drift rate)

$$\mu_1 = h(W,\sigma) \cdot [R_I - \exp(-\delta_1 \cdot T_D) \cdot R_D]$$

Implementation into dynamic stochastic dual process framework: System 2

- $E[V_{2_I}(t)] = R_I \cdot t$ for the immediate reward
- $E[V_{2_D}(t)] = \exp(-\delta_2 \cdot T_D) \cdot R_D \cdot t$ for the delayed reward, resulting in a mean difference in valence (which defines the drift rate)

$$\mu_2 = R_I - \exp(-\delta_2 \cdot T_D) \cdot R_D.$$

Stimulus values and parameters

Situation	R_I	R_D	T_D	δ_1	δ_2
1	5.5	9.35	15	.9	1.1
2	3	3.3	30	.9	1.1

Subset of stimulus values used by Zhao et al. (2019).

R_I: immediate rewards

R_D: the delayed rewards

T_D: time delay

δ_1: discount factor in System 1

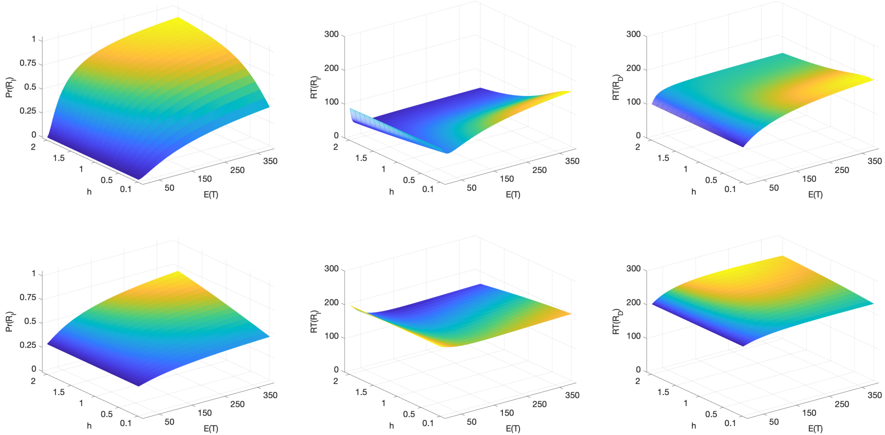
δ_2: discount factor in System 2

$\delta_1 < \delta_2$

- $h(W, \sigma) = h$, a constant

Two-stage dynamic dual process model predictions

- The longer the DM operates in System 1, the stronger the myopic effect.
- Preference reversal



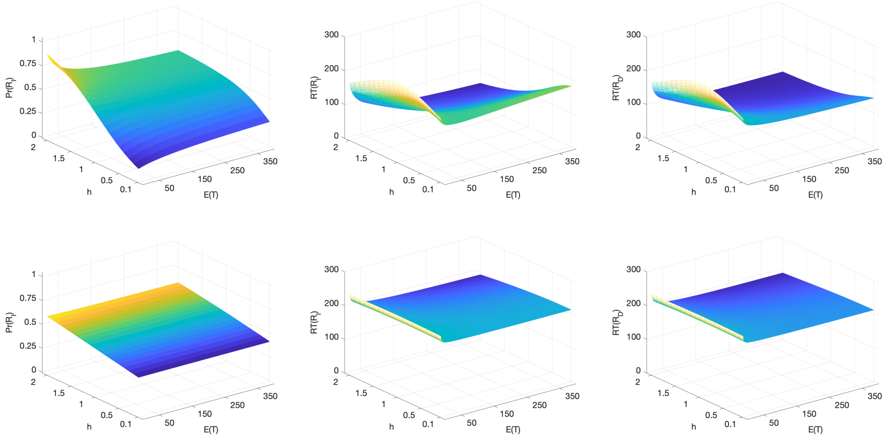
Predictions RT

Given that the mean valences (drift rates) in both systems point in opposite directions, then the following patterns hold:

- Preference reverses with increasing time spent operating in System 1. That is, the probabilities for choosing one alternative change from below (above) 0.5 to above (below) 0.5 as operating time in System 1 increases.
- The larger the mean valence in System 1 is as compared to System 2 (absolute values), the sooner the reversal occurs as a function of operating time in System 1.
- When the mean valence in System 1 is larger (absolute value) than in System 2, then the more frequently chosen alternative is faster after the preferences reversal.
- When the mean valence in System 1 is smaller than in System 2, then the less frequently chosen alternative is faster after the preferences reversal.

Basic Parallel dynamic dual process model predictions

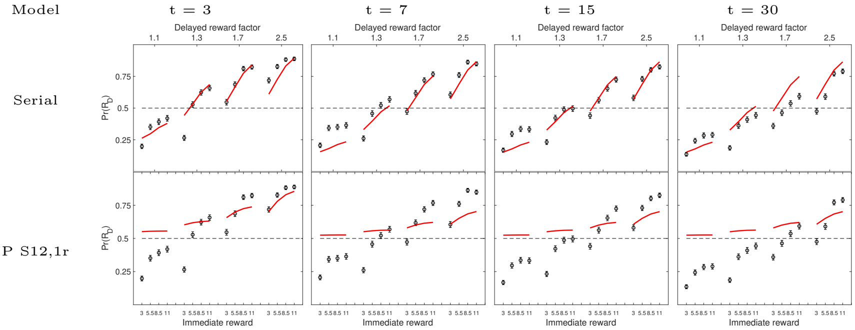
- The time the DM operates in System 1 and System 2 has no/little effect on the size of the myopic effect .
- No preference reversal



Testing the model

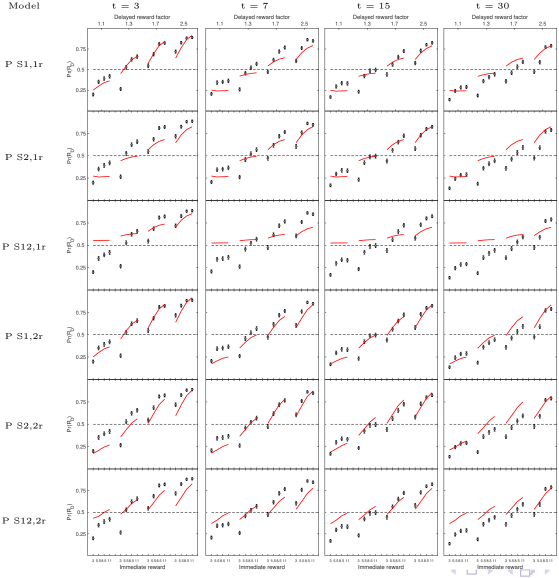
- Zhao et al. (2019) data
- 51 participants
- Immediate rewards: \$3, \$5.5, \$ 8.5, and \$11
- T_D : 3, 7, 15, and 30 days
- Multipliers for the delayed reward: 1.1, 1.3, 1.7, and 2.5
- 64 different choice pairs (4 immediate reward $\times$ 4 delayed reward factors $\times$ 4 time delays)
- 10 presentations of each choice pair
- Each choice proportion is based on 504 observations.

Model accounts



Serial model version (upper row) and the basic quasi-parallel model version, i.e., assuming an equal lead for any of the two systems (S12) and the same distribution and expected value $E(T) = 1/r_2 = 1/r_2$ of operating time in both systems (1r).

Model accounts: parallel versions

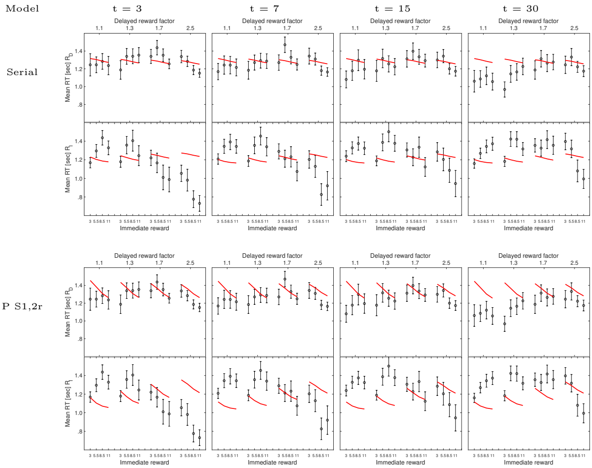


Model comparison – goodness of fit

Model	start with System	$E(T)$	#Parameters	AIC
S	1	$1/r_1$	5	1663
P	1	$1/r_1 = 1/r_2$	5	2516
P	2	$1/r_1 = 1/r_2$	5	3303
P	any	$1/r_1 = 1/r_2$	5	10567
P	1	$1/r_1 \neq 1/r_2$	6	1460
P	2	$1/r_1 \neq 1/r_2$	6	2188
P	any	$1/r_1 \neq 1/r_2$	6	4656

AIC for the 7 fitted model versions (serial (S) and 6 parallel (P)). For comparison of the parallel models, the time schedule was fixed and did not count as an additional parameter. For a proper comparison with the serial model version, the number of parameters of the parallel versions increases by 1.

Response times



Observed and predicted mean choice reposes times for the two best fitting models. Note that only 1 parameter, RT_{res}, was estimated from 128 data points. AIC = 1241 for the serial version; AIC = 2311 for the quasi-parallel version.

Concluding remarks

- Most dual-process models proposed so far are verbal descriptions.
- Most intertemporal choice models are static and deterministic.
- The dynamic dual-process model captures the stochastic and dynamic nature of decision making.
- Two-stage processes outperform single processes (not shown).
- The two-stage dual process model and the quasi-parallel dual process model make distinct predictions → help solving the controversy between "default-interventionist" and "parallel-competitive" approach
- Independent race (parallel processing): makes the notion of a faster System 1 redundant (always the winner)
- Mixture of two systems: makes the notion of two interacting processes operating together redundant.

Thank you

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Predictions RT

Given that the mean valences (drift rates) in both systems do not point in different directions then the following patterns hold:

- If the mean valence in System 1 is larger (in absolute value) than in System 2 ($0 \leq \mu_2 < \mu_1$ or $\mu_1 < \mu_2 < 0$), then then the two-stage model **always** predicts faster mean response time to the more frequently chosen alternative.
- If the mean valence in System 1 is smaller (in absolute value) than in System 2 ($0 \leq \mu_1 < \mu_2$ or $\mu_2 < \mu_1 < 0$), then the two-stage model **always** predicts faster mean response times to the less frequently chosen alternative.